A Parallel Implementation of dual-PEEC for Multicore and Multithreaded CPUs

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***Abstract*—The Partial Element Equivalent Circuit, as well as other integral techniques, is characterized by full matrices, whose assembly phase is not negligible in terms of computational cost. The aim of the work is to analyze the bottlenecks of the such types of codes and propose an efficient implementation suitable for modern multicore and multithreaded architectures.**

***Index Terms*—Parallel computing, Partial element equivalent circuit, Dual discretization**

I. Introduction

The Partial Element Equivalent Circuit (PEEC), is a well assessed integral technique whose origins go back to the middle '70s in the seminal work by Ruehli [1] and which is still receiving considerable attention [2], [3]. The basic idea is represent an electromagnetic structure by means of an equivalent lumped circuit which can then be solved by SPICE-like circuit simulators in the time or frequency domain. The starting point is the equation of the total electric field which, at the observation point $\vec{r}$ and angular frequency ω, takes the form:

$$\frac{\vec{J}(\vec{r},\omega)}{\sigma} = -\mathrm{j}\omega\vec{A}(\vec{r},\omega) - \nabla\varphi(\vec{r},\omega) \tag{1}$$

where $\vec{J}$ is the current density and $\vec{A}$ and φ are the retarded magnetic vector and electric scalar potential, respectively, which can be expressed as:

$$\vec{A}(\vec{r},\omega) = \frac{\mu}{4\pi}\int_\Omega \frac{\vec{J}(\vec{r}',\omega)}{|\vec{r}-\vec{r}'|}\mathrm{e}^{-\mathrm{j}\beta|\vec{r}-\vec{r}'|}\mathrm{d}\Omega$$

$$\varphi(\vec{r},\omega) = \frac{1}{4\pi\epsilon}\int_\Omega \frac{\rho(\vec{r}',\omega)}{|\vec{r}-\vec{r}'|}\mathrm{e}^{-\mathrm{j}\beta|\vec{r}-\vec{r}'|}\mathrm{d}\Omega$$

where ρ is the volume charge density, β is the wavenumber, $\vec{r}'$ the source point and Ω the conductor domain. Standard-PEEC uses pulse basis functions both for current density and for charge density but with different support and the same basis functions are also used as weights in a Galerkin scheme. In the case of non-orthogonal PEEC discretization [4], numerical integration is required for the calculation of partial coefficients [5].

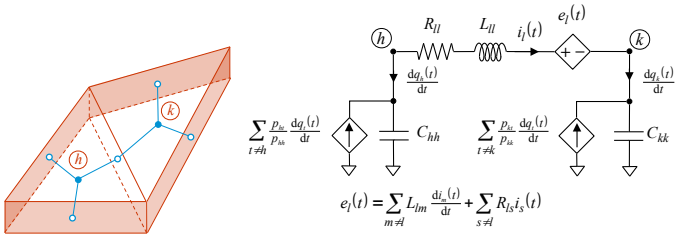
II. Dual-PEEC formulation

Dual-PEEC is a generalization of PEEC to unstructured triangular tessellations [6]. The scheme makes use of the topological concept of *duality* so that physical variables are associated to spatial elements as proposed by Tonti and shown in Table I. According to such scheme the current density, flowing through the triangles' sides, is interpolated by "face" elements

$$\vec{J}_m = \frac{1}{t_m}\sum_{k=1}^{3}\vec{w}_{km}i_{km}$$

TABLE I
Variables and their association to spatial entities.

Type	Variable	Spatial Element
source	current, i	dual face, $\tilde{\Sigma}$
	charge, q	dual volume, $\tilde{\Omega}$
configuration	voltage, u	primal edge, λ
	potential, φ	primal node P



(a) (b)

Fig. 1. Basic geometric entities involved in the discretization and equivalent circuit.

while the charge associated to triangles is represented by pulse functions (see Fig.1(a)). The couple constituted by primal edge/dual face is associated to a circuit branch characterized by:

- resistive coupling (neglected in this study)
- inductive coupling

$$L_{km,hn} = \frac{\mu_0}{4\pi}\int_{\Sigma_m}\vec{w}_{km}\cdot\int_{\Sigma_n}\frac{\vec{w}_{hn}\mathrm{e}^{-\mathrm{j}\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\mathrm{d}\Sigma_n\mathrm{d}\Sigma_m \tag{2}$$

- capacitive coupling

$$p_{m,n} = \frac{1}{4\pi\epsilon_0 S_m S_n}\int_{\Sigma_m}\int_{\Sigma_n}\frac{\mathrm{e}^{-\mathrm{j}\beta|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|}\mathrm{d}\Sigma_n\mathrm{d}\Sigma_m \tag{3}$$

The final resulting basic circuit block is shown in Fig. 1(b).

III. Partial element calculation

The assembly of the matrices representing the inductive and capacitive coupling requires the computation of the integrals in (2) and (3). When an orthogonal discretization is possible, the computational burden of the partial elements computation is

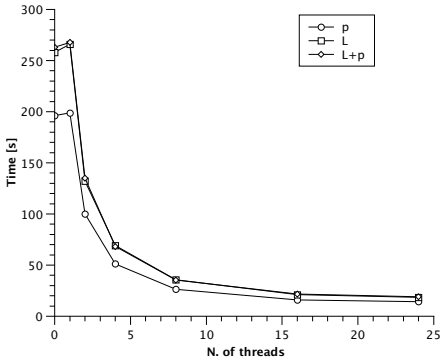


Fig. 2. CPU time for a 10.000 element mesh.

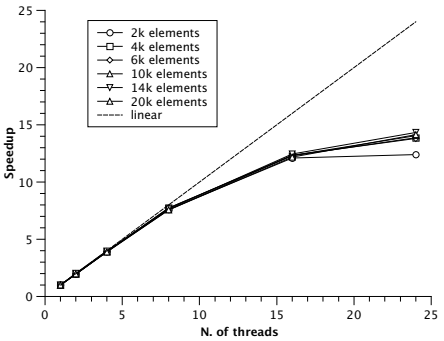


Fig. 3. Speedup for various mesh sizes.

negligible with respect to solution time, however with non-orthogonal or triangular discretization, the partial element computational cost can become comparable with the solution time [7]. Partial elements with triangular discretization are computed with the following considerations:

- analytical formulas exist for the solution of the inner integral both for inductance and for coefficients of potential [8]
- the outer integral is computed by appropriate quadrature rules on triangles (16 or 25 points)
- since the integration domain is the same for inductances and coefficients of potential, the evaluation of the latter can be obtained at negligible cost as shown in Fig. 2.

The integration routines have been parallelized with the help of OpenMP at the coarsest level, namely all elements are processed in parallel.

IV. Results

Computational results are obtained on a Dell T7500 equipped with dual Intel Xeon X5670 (each processor has 6 cores/12 threads) running at 2.93 GHz with 12Mb cache and a total shared memory of 96Gb. The machine runs Fedora 13 64bit (Kernel 2.6.33 x86_64) and the code was compiled with the Intel Fortran Compiler 11.1 with flags: `-O3 -m64 -fast -openmp`.

Table II and Fig. 3 show the kind of speedup obtainable on the target architecture. The same table demonstrates shows that the code performs in accordance with Amdahl's law $T_{\text{elapsed}}(P) = \left(\frac{f}{P} + 1 - f + O_{\text{P}}P\right) T_{\text{serial}}$ in which P is the number of threads and f is the fraction of parallelized code. It can be seen that the implementation is optimal in the sense that $f = 1$ and the speedup saturation is entirely due to the communication overhead O_{P} caused by the different processes writing to the shared memory area containing the full matrices.

TABLE II
Computed and estimated parallel performance.

# of threads	Comp. time (s) measured	Amdahl's law (s) $f = 1$, $O_{\text{P}} = .0013$
serial	263.14	-
1	267.69	263.48
2	134.97	132.39
4	68.27	67.35
8	35.41	35.86
16	21.73	22.17
24	19.05	19.43
26	-	19.27
28	-	19.23
30	-	19.29

f: parallel fraction of code, O_{P}: parallel overhead, P: # of processors

V. Conclusions

Dual-PEEC allows the joint calculation of inductance and coefficient of potentials, reducing the overall computational effort with respect to standard nonorthogonal-PEEC. Furthermore, the computation of coefficients has been parallelized with OpenMP showing good scalability, according to Amdahl's law. In the extended version of the paper it will be shown that additional speedup can be obtained by suitable hierarchic quadrature rules, decreasing the number of quadrature points for weakly interacting elements. Furthermore the recent IBM Power7 architecture will also be evaluated in order to assess if the communication overhead can be reduced.

References

[1] A. Ruehli, "Equivalent circuit models for three-dimensional multiconductor systems," *IEEE Transactions on Microwave Theory and Techniques*, vol. 22, pp. 216–221, 1974.

[2] P. Scholz, W. Ackermann, T. Weiland, and C. Reinhold, "Antenna modeling for inductive RFID applications using the partial element equivalent circuit method," *IEEE Transactions on Magnetics*, vol. 46, no. 8, pp. 2967–2970, 2010.

[3] W. Yahyaoui, L. Pichon, and F. Duval, "A 3D PEEC method for the prediction of radiated fields from automotive cables," *IEEE Transactions on Magnetics*, vol. 46, no. 8, pp. 3053–3056, 2010.

[4] A. Ruehli, G. Antonini, J. Esch, J. Ekman, A. Mayo, and A. Orlandi, "Nonorthogonal PEEC formulation for time- and frequency-domain EM and circuit modeling," *IEEE Transactions on Electromagnetic Compatibility*, vol. 45, no. 2, pp. 167 – 176, 2003.

[5] A. Musing, J. Ekman, and J. Kolar, "Efficient calculation of non-orthogonal partial elements for the PEEC method," *IEEE Transactions on Magnetics*, vol. 45, no. 3, pp. 1140–1143, 2009.

[6] F. Freschi, L. Giaccone, and M. Repetto, "Educational value of the algebraic numerical methods in electromagnetism," *COMPEL - The International Journal for Computation and Mathematics in Electrical and Electronic Engineering*, vol. 27, no. 6, pp. 1343–1357, 2008.

[7] J. Ekman and P. Anttu, "Parallel implementations of the peec method," in *Proceedings of IEEE International Symposium on Electromagnetic Compatibility*, Jul 9–13 2007, pp. 1–6.

[8] R. Graglia, "On the numerical integration of the linear shape functions times the 3-D Green's function or its gradient on a plane triangle," *IEEE Transactions on Antennas and Propagation*, vol. 41, pp. 1448–1455, 1993.